Overview — Tuesday, Aug 28 2001

- Spatial Inference
  ... in SPM
- Combining Results
  ... fixed and random effects

Spatial Inference: Recap

- We fit a model at every point in the brain
  - ⇀ Each model is ignorant of neighboring models
- Each model summarized with a t or F statistics
- Images of these statistics are thresholded
  - ⇀ Set-, Cluster- & Voxel-level p-values assigned
- "Implicit Spatial Modeling"

Spatial Inference: SPM

- Important to understand each component of SPM p-value results page
- Top
  - ⇀ Interactive MIP
    - – Drag & Drop to move
    - – Right click to get menu
  - ⇀ Interactive Design Matrix
    - – Click to get values
- Middle
  - ⇀ Interactive table of corrected & uncorrected p-values
    - – Click on x,y,z to move cursor
    - – Click on number to show it in Matlab window
    - – Right click in margin to get menu

Spatial Inference: SPM

- Bottom
  - ⇀ Details on results
  - ⇀ Key key key!
    - – Smoothness must be $>\approx$ 2-3 voxel size
    - – This is needed for

      ______________________ to be valid

Spatial Inference: SPM etc

- `SPM.mat`
  - ⇀ Contains essentials of fitted model results
  - ⇀ Design matrix, number of voxels, etc
- `SPMcfg.mat`
  - ⇀ "SPM Configuration file"
  - ⇀ Input to 'Estimate' button
  - ⇀ Specifies everything necessary to fit a model

Spatial Inference: SPM etc

- `SPM_fMRIDesMtx.mat`
  - ⇀ Specification of fMRI design
  - ⇀ Does *not* contain any filenames
  - ⇀ If identical design is used, can copy these into other analysis directories
  - ⇀ For example
    - – I could have created one `SPM_fMRIDesMtx.mat` that the whole class could use, since all of our subject's had identical paradigms
- "Explore Design button works with any of these.

Combining Results

- Often have results we want to combine
  - ⇀ Same subject, different contrasts
  - ⇀ Same subject, different acquisition
  - ⇀ Same subject, different days
  - ⇀ Different subjects
- How to integrate these results?
- Two fundamentally different approaches
  - ⇀ Fixed Effects
  - ⇀ Mixed/Random Effects
    - – "Random Effects" has become standard, though strictly it is a Mixed Effects model

Combining Results: Fixed Effects

- Fixed effects test an intersection of null hypotheses
  - ⇀ $\mathcal{H}_0$: The effect zero in *all* sessions
  - ⇀ $\mathcal{H}_0$: The effect zero in *all* subjects
- Fixed effects hypotheses are easily tested
  - ⇀ Chuck all the data into SPM
  - ⇀ Test the grand hypothesis
    - – Example: 4 subjects, each with 2 conditions

The contrast of interest is then

Combining Results: Fixed Effects

- "Grand" model approach not perfect
  - ⇀ Can be huge, slow analysis
    - – Thousands of scans!
    - – Tons of disk space
  - ⇀ Assumes homogeneous variance across subjects
- But can be very sensitive

Combining Results: Fixed Effects

- Simpler approach is to combine statistic images
- Metaanalysis approach
  - ⇀ K analyses, each produces $Z_k, k = 1, ..., K$
  - ⇀ Under null hypothesis, each $Z_k \sim \mathcal{N}(0, 1)$
  - ⇀ $\bar{Z} \sim \mathcal{N}(0, 1/K)$
  - ⇀ So summarize K statistics with $\sqrt{K}\bar{Z} = \frac{1}{\sqrt{K}} \sum Z_k$
- Weaker assumptions than Grand Model approach
  - ⇀ Only have to assume that each individual model is valid
  - ⇀ (Not that all have common variance)
- Unfortunately, not supported within SPM
  - ⇀ Can calculate with "ImCalc"
  - ⇀ But can't get table of p-values

Combining Results: Fixed Effects Shortcomings

- Doesn't account for intersubject variability in response
  - ⇀ Significance can be due to a single subject
- Susceptible to hypothesis testing facility
  - ⇀ Given enough data, you'll *always* reject the null!
  - ⇀ This makes sense.
  - ⇀ Do you ever really believe "$\mathcal{H}_0$: Effect is $0.000000000000\bar{0}$" ?

Combining Results: Conjunction of Contrasts

- Tries to address weakness of fixed effects approach
- Standard fixed effects approach can be driven by one "best" subject/contrast/effect
- SPM's Conjunction
  - ⇀ Take the *minimum* of multiple statistics
  - ⇀ Then only can see effect if *all* large
- Example
  - ⇀ 4 subjects, each with 2 conditions
  - ⇀ Create 4 contrasts, one for each subject
  - ⇀ "Conjunction" of the four is just minimum of four statistic images

Combining Results: Conjunction of Contrasts

- SPM Conjunction
  - ⇀ Random field theory requires individual contrasts to be orthogonal
  - ⇀ Two contrasts orthogonal if *inner product* is zero
  - ⇀ These are orthogonal: [-1 1 0 0] & [0 0 -1 1]
  - ⇀ These are not: [0 -1 1 0] & [0 0 -1 1]

Combining Results: Contrast Masking

- Can combine contrast/statistic images with masks
- Example... Two factor study
  - ⇀ Faces vs Places (A1 vs A2)
  - ⇀ Familiar vs Unfamiliar (B1 vs B2)
  - ⇀ In SPM

    [A1 A2 B1 B3]

Combining Results: Contrast Masking

- To view familiarity effect *but only* where there was a Faces effect...
  - ⇀ Select "Familiar-Unfamiliar" in contrast manager
  - ⇀ "Yes" to "mask with other contrast"
    - – Specify "Faces-Places" contrast
  - ⇀ Set "uncorrected mask p-value"
    - – e.g. 0.001
  - ⇀ Select "inclusive" mask
    - – Want to see voxels where Faces-Places > threshold
  - ⇀ *et voila*
- No account of thresholding in subsequent inference
  - ⇀ But only would increase significance

Combining Results: Random Effects

- What is a "random effect"?
  - ⇀ Well, have one already

    $$Y = X\beta + \epsilon$$

- Random effect models treat *response magnitude* as random
  - ⇀ So far $X\beta$, i.e. $c\beta$ has been *fixed*!
  - ⇀ Only randomness is residual or measurement error

Combining Results: Visualizing Random Effects

Fixed Effect

Random Effects

Combining Results: Understanding Random Effects

- With fixed effects
  - ⇀ Combing over subjects, only question is

    "Is average response large relative to scan-to-scan variability"

- With random effects
  - ⇀ Combing over subjects, question is

    "Is average response large relative to subject-to-subject variability"

Combining Results: Understanding Random Effects

- Fixed Effects Model & Null Hypothesis
  - $Y(t) = \mu + \alpha f(t) + \epsilon(t)$
  - Fixed, known: $f(t)$
  - Fixed, unknown: μ, α
  - Random: $\epsilon(t)$
  - Dist^n assumption: $\epsilon(t) \sim \mathcal{N}(0, \sigma^2)$
  - $\mathcal{H}_0$: $\mu = 0$

    Mean of each subject's response is zero

Combining Results: Understanding Random Effects

- Random Effects Model & Null Hypothesis
  - $Y_i(t) = \mu_i + \alpha_i f(t) + \epsilon_i(t)$
  - Fixed, known: $f(t)$
  - Fixed, unknown: μ_i
  - Random: $\alpha_i, \epsilon_i(t)$
  - Dist^n assumption: $\epsilon_i(t) \sim \mathcal{N}(0, \sigma^2)$

    $\alpha_i \sim \mathcal{N}(\theta, \tau^2)$

    $\epsilon_i(t), \alpha_i$ independent
  - $\mathcal{H}_0$: $\theta = 0$

    Mean of the *population* of the subject's response is zero

Combining Results: Random Effects in SPM

- SPM only fits fixed effects models
  - In general, random effects models can't be fit with least squares
  - Generally requires iterative fitting
- One special *very useful* case
  - Assume each subject has same experimental design
    - Same number of scans, same design matrix
  - Assume each subject's residual error (σ^2) is the same
  - Fit each subject, create a contrast for each
  - Compute a one sample t test on contrast images
- This approach impliments random effects inference!

Combining Results: Random Effects in SPM

- What if not all have same design matrix
  - An issue of a balanced design
    - If not too unbalanced, probably OK
  - If way unbalanced, probably way dodgy

Combining Results: Random Effects in SPM

- Example
  - Places vs Faces
  - After study, subjects asked to score familiarity of images
  - Result used to define dichotomous Familiar/Nonfamiliar factor
  - Interest is in Places-Faces only for Familiar stimuli
  - Problem!
    - If some subjects find only 10% familiar, and other find 90%

      there will be horrible imbalnce in the design matrix
    - Put another way, some subjects will be able to offer 90% of their data to the contrast, others only 10%

Inference Quiz

I-1 What is the most important value in the footer of SPM's results display?

I-2 What is you favorite color? (Blue, no...) Er, no, What is your favorite hidden trick on the SPM results page?

I-3 Are these two contrasts orthgonal?

[0 1 0 -1]

[-1 0 -1 0]

I-4 Are these two contrasts orthgonal?

[1 1 0 1 1 -1]

[1 0 -1 1 -1 1]

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